Experimental requirements to determine the neutrino mass hierarchy using reactor neutrinos

Liang Zhan, Yifang Wang, Jun Cao, and Liangjian Wen
Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
(Received 20 January 2009; published 14 April 2009)

This paper presents experimental requirements to determine the neutrino mass hierarchy using reactor neutrinos. The detector shall be located at a baseline around 58 km from the reactor(s) to measure the energy spectrum of electron antineutrinos ($\overline{\nu}_e$) precisely. By applying Fourier cosine and sine transforms to the L/E spectrum, features of the neutrino mass hierarchy can be extracted from the $|\Delta m^2_{31}|$ and $|\Delta m^2_{32}|$ oscillations. To determine the neutrino mass hierarchy above 90% probability, requirements to the baseline, the energy resolution, the energy scale uncertainty, the detector mass, and the event statistics are studied at different values of $\sin^2(2\theta_{13})$.

DOI: 10.1103/PhysRevD.79.073007 PACS numbers: 13.15.+g, 14.60.Lm, 14.60.Pq

Neutrino physics has undergone a revolution over the last decade and reaches now to an era of precision measurement for the neutrino oscillation parameters. However, θ_{13}, CP-violating phase, and the sign of Δm^2_{32} (mass hierarchy) are still undetermined. Usually normal hierarchy (NH) is defined as $\Delta m^2_{32} > 0$ and inverted hierarchy (IH) is defined as $\Delta m^2_{32} < 0$. Accelerator neutrino experiments such as Nova [1–3] and T2KK [4] have the potential to determine the mass hierarchy using the matter effect of neutrinos at long baselines. There are also discussions to precisely measure the distortions of the reactor neutrino energy spectrum at an intermediate baseline (40–65 km) [5,6]. A Fourier transform method was recently proposed to enhance and visualize the features of mass hierarchy in the frequency (Δm^2) spectrum [7].

A new study [8] based on Fourier transformation utilizing both the amplitude and phase information is presented recently to enhance the features distinguishing the mass hierarchy at a very small $\sin^2(2\theta_{13})$ value. This paper is complementary to Ref. [8] by taking into account experimental details, such as the baseline, detector response including energy resolution, energy scale uncertainty, and the event statistics, etc. The study is based on Monte Carlo simulation.

Taking into account the detector response, the reactor neutrino $\overline{\nu}_e$ L/E spectrum F(L/E) becomes

$$F(L/E') = \int R(E, E')F(L/E)\mathrm{d}E, \quad F(L/E) = \phi(E)\sigma(E)P_{ee}(L/E), \tag{1}$$

where L is the baseline, E is the actual $\overline{\nu}_e$ energy, E' is the observed $\overline{\nu}_e$ energy taking into account the detector response, and $R(E, E')$ represents the detector response including effects such as the energy resolution and energy scale. The reactor neutrino flux, $\phi(E)$, the neutrino inverse beta reaction cross section with detector, $\sigma(E)$, and the neutrino oscillation probability, $P_{ee}(E)$, have all been described in Ref. [8]. Here for completeness, we rewrite the $\overline{\nu}_e$ survival probability $P_{ee}(E)$ [9] as

$$\begin{aligned} P_{ee}(L/E) &= 1 - P_{21} - P_{31} - P_{32}, \\ P_{21} &= \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2(\Delta_{21}), \\ P_{31} &= \cos^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{31}), \\ P_{32} &= \sin^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{32}). \end{aligned} \tag{2}$$

The analytical formulas for Fourier cosine and sine transform [8] are

$$\mathrm{FCT}(\omega) = \int_{t_{\min}}^{t_{\max}} F(t)\cos(\omega t)dt, \quad \mathrm{FST}(\omega) = \int_{t_{\min}}^{t_{\max}} F(t)\sin(\omega t)dt, \tag{3}$$

where ω is the frequency defined as $2.54\Delta m^2$; $t = L/E$ is the variable in L/E space, varying from $t_{\min} = L/E_{\max}$ to $t_{\max} = L/E_{\min}$. In real experiments with a set of discrete events, the integral can be changed to the summation over all events as in the following:

$$\mathrm{FST}(\omega) = \sum_{i=1}^{N} \sin(\omega L/E'_i), \quad \mathrm{FCT}(\omega) = \sum_{i=1}^{N} \cos(\omega L/E'_i), \tag{4}$$

where E'_i is the measured energy of individual events, and N is the total number of events collected.

The actual experimental measurements of the neutrino energy usually have two aspects of detector responses: energy resolution and energy scale. The response of the detector due to energy resolution can usually be described by a Gaussian function $\frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{(E'-E)^2}{2\sigma_E^2})$, where σ_E is the energy resolution. Since the neutrino energy are usually measured by scintillators, the energy is typically proportional to the number of photoelectrons, and the error is dominated by the photoelectron statistics. Therefore, the neutrino energy resolution is proportional to $1/\sqrt{E_{\mathrm{vis}}}$, where $E_{\mathrm{vis}} = E_\nu - 0.8$ MeV is the neutrino visible energy in the detector. Previous experiments typically have an

TABLE I. Default values for neutrino oscillation parameters and other input parameters studied in this paper.

Δm^2_{21}	$\lvert\Delta m^2_{32}\rvert$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$
7.6×10^{-5} eV^2	2.4×10^{-3} eV^2	0.32	0.50

$\sin^2(2\theta_{13})$	L	σ_E	a	b
0.02	58 km	$3\%/\sqrt{E_{\rm vis}}$	1%	0.01 MeV

energy resolution of about $10\%/\sqrt{E_{\rm vis}}$. Different detectors may have different forms of the energy scale uncertainty. For simplicity, we take two possible cases, shift and shrinking/expanding. It is modeled as formula $E' = (1+a)E + b$, where a and b are parameters.

In this study, each Monte Carlo experiment generates a set of $\overline{\nu}_e$ events by sampling the F(L/E) spectrum with input parameters $\{\sin^2\theta_{13}, L, \sigma_E, a, b\}$. The total number of generated events determines the statistical error. Default oscillation parameters are taken from Ref. [8] and reproduced here in Table I, together with default input parameters to be studied in this paper.

In the following study, effects of input parameters to the mass hierarchy are studied one by one, while the rest remain at the default values. It is known that at the oscillation maximum of Δm^2_{12}, corresponding to a baseline of about 58 km, the sensitivity to the mass hierarchy is maximized. Hence the default value of the baseline is set at 58 km. The mass hierarchy effects in the reactor neutrino energy spectrum are mainly characterized by $\Delta m^2_{21}/\lvert\Delta m^2_{32}\rvert$, which is only about 3%. Hence the required energy resolution shall be at the level of this number and the default value in this study is set to be $3\%/\sqrt{E_{\rm vis}}$. Previous experiments show that a and b are typically at the level of 1% and 0.01 MeV, respectively. The observed neutrino event number is proportional to the detector volume, exposure time, and reactor(s) power. A very powerful reactor complex can consist of 8 reactor cores, each with $\sim$3 GW thermal power. With a baseline of 58 km from such a reactor complex and taking into account the oscillation probability which is around $\sin^2 2\theta_{12}$, 5×10^5 events, corresponding to a detector exposure of $\sim$700 kt $\cdot$ year, is taken as the default for an experiment.

Figure 1 shows FCT and FST spectra from a Monte Carlo simulation using parameters $(\sin^2(2\theta_{13}), \sigma_E) = (0.02, 3\%/\sqrt{E_{\rm vis}})$. For comparison, the analytical spectra are also shown at $\sin^2(2\theta_{13}) = 0.02$. The impacts of the energy resolution and statistical errors are obviously seen as that the amplitudes of noisy peaks and valleys appear to be higher in the frequency range of 2.0×10^{-3} eV$^2 < \Delta m^2 < 2.8\times10^{-3}$ eV^2. However, the main peak and valley are distinctive and can still be used to determine the neutrino mass hierarchy.

We introduce parameters RL and PV [8] to quantify the features of FCT and FST spectra.

$$\mathrm{RL} = \frac{\mathrm{RV}-\mathrm{LV}}{\mathrm{RV}+\mathrm{LV}}, \qquad \mathrm{PV} = \frac{\mathrm{P}-\mathrm{V}}{\mathrm{P}+\mathrm{V}} \tag{5}$$

where RV is the amplitude of the right valley and LV is that of the left valley in the FCT spectrum; P is the amplitude of the peak and V is that of the valley in the FST spectrum.

For each set of input parameters $\{\sin^2\theta_{13}, L, \sigma_E, a, b, N\}$, we simulate 500 experiments and calculate the probability to determine the mass hierarchy based on the distributions of RL and PV values. The procedure is concluded as the following:

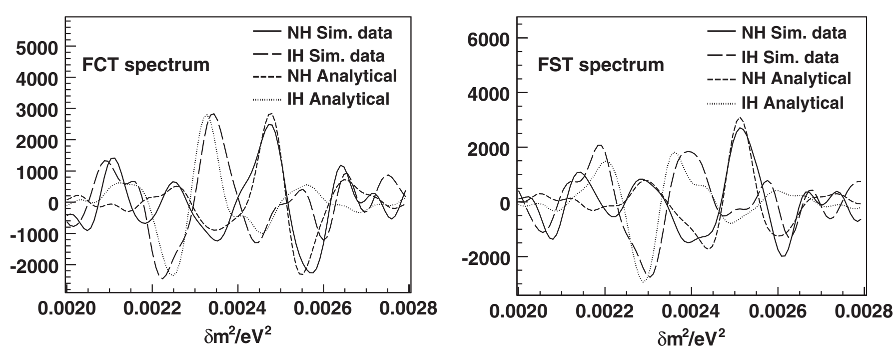
(1) Given $\sin^2\theta_{13}$ and L, we sample N neutrino events with energy $E_i (i = 1, 2, \ldots, N)$ from energy spectrum both for NH and IH cases.

(2) E_i is smeared and/or shifted to E'_i based on the given energy resolution (σ_E) and energy scale uncertainty (a and b) parameters.

(3) FCT and FST spectra are calculated using Eq. (4).

(4) RL and PV values are calculated based on FCT and FST spectra using Eq. (5).

(5) Repeat the above steps 500 times and obtain the distributions of RL and PV values.



(a) FCT spectrum. (b) FST spectrum.

FIG. 1. FCT and FST spectra from simulation with parameters $(\sin^2(2\theta_{13}), \sigma_E) = (0.02, 3\%/\sqrt{E_{\rm vis}})$, together with the analytical spectra for $\sin^2(2\theta_{13}) = 0.02$. Solid and long-dashed lines are spectra based on simulation for NH and IH cases, while dashed and dotted lines are analytical spectra.

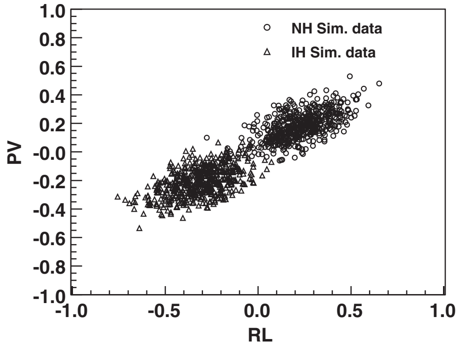


FIG. 2. Distribution of RL and PV values from 500 simulated experiments with parameters $(\sin^2(2\theta_{13}), \sigma_E) = (0.02, 2\%/\sqrt{E_{\rm vis}})$. Two clusters of points show the sensitivity to determine the mass hierarchy.

(6) Calculate the probability to determine the mass hierarchy correctly based on the distributions of RL and PV values.

Figure 2 shows the distribution of RL and PV values for 500 experiments with input parameters $(\sin^2(2\theta_{13}), \sigma_E) = (0.02, 2\%/\sqrt{E_{\rm vis}})$. Two clusters of points in the (RL, PV) plane corresponding to NH and IH cases show the probability to determine the mass hierarchy. Various input parameters have been tried and the distribution of RL + PV is shown in Fig. 3. Two clusters of points turn into two Gaussian distributions and the probability to determine the mass hierarchy can be correctly calculated.

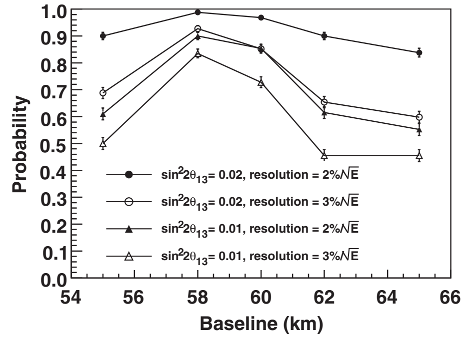


FIG. 4. Impact of the baseline to the determination probability for 4 sets of parameters $(\sin^2(2\theta_{13}), \sigma_E)$.

To study the impact of the baseline, a total of 500 experiments have been simulated for each set of input parameters $(\sin^2(2\theta_{13}), \sigma_E)$. Figure 4 shows the results. The error bars are due to statistics since only a limited number of experiments are simulated. The optimal baseline is clearly 58 km, which is chosen as the default baseline.

Figure 5 shows the impact of event number to the determination probability. Obviously, fewer number of events will induce larger statistical fluctuations, more noisy

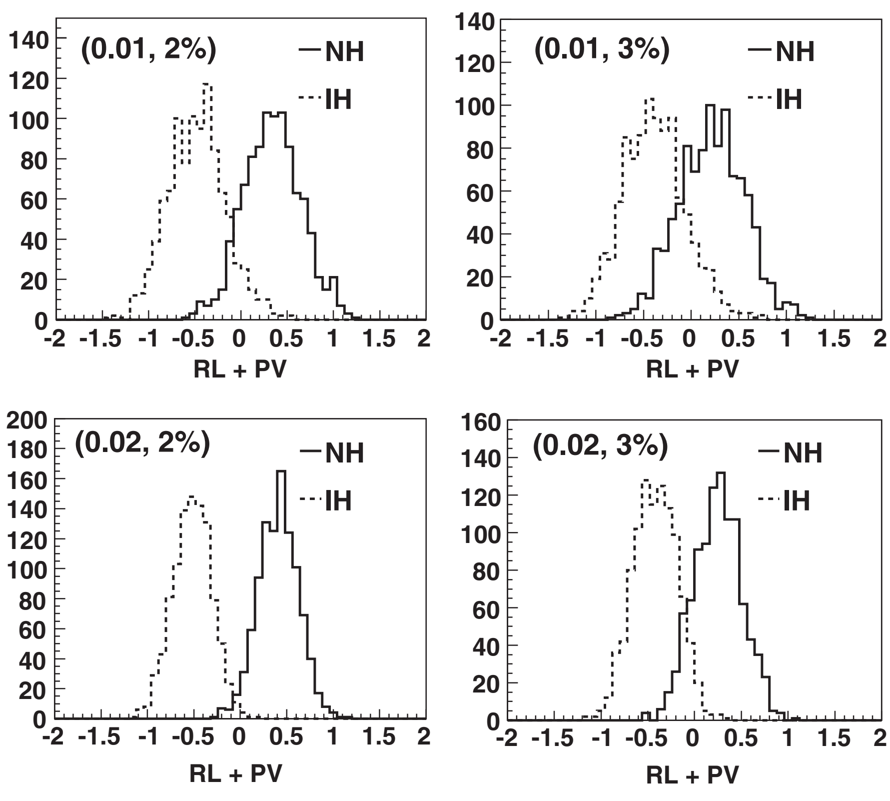


FIG. 3. Distribution of RL + PV values from 1000 experiments for parameters $(\sin^2(2\theta_{13}), \sigma_E)$ being $(0.01, 2\%/\sqrt{E_{\rm vis}})$, $(0.01, 3\%/\sqrt{E_{\rm vis}})$, $(0.02, 2\%/\sqrt{E_{\rm vis}})$, and $(0.02, 3\%/\sqrt{E_{\rm vis}})$. Two Gaussian distributions show the sensitivity to determine the mass hierarchy.

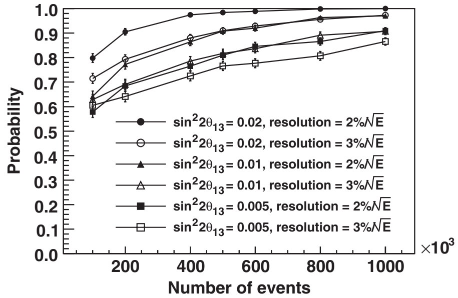


FIG. 5. Impact of the event number to the determination probability in six sets of parameters ($\sin^2(2\theta_{13})$, σ_E).

peaks and valleys in the FCT and FST spectra and hence reduce determination probability. As shown in Fig. 5, a total of 5×10^5 events will reach 90% determination probability for $\sin^2(2\theta_{13}) = 0.02$ with an energy resolution of $3\%/\sqrt{E_{\rm vis}}$. This number of events, probably the largest can be imagined nowadays, is chosen as the default in this paper.

The requirement to the event statistics strongly depends on the value of $\sin^2(2\theta_{13})$. Figure 6 shows the number of neutrino events needed to determine the mass hierarchy at the 90% confidence level as a function of $\sin^2(2\theta_{13})$. Two cases of the energy resolution, $2\%/\sqrt{E_{\rm vis}}$ and $3\%/\sqrt{E_{\rm vis}}$, are studied. If $\sin^2(2\theta_{13})$ happens to be more than 0.05, as some of the recent global fit indicated [10], the number of events can be a factor of 5 smaller than that in the case of $\sin^2(2\theta_{13}) = 0.02$.

Impact of the energy resolution to the mass hierarchy determination is studied for the cases of $\sin^2(2\theta_{13}) = 0.02$, 0.01, and 0.005 as shown in Fig. 7. To achieve the mass hierarchy determination probability better than 90% at $\sin^2(2\theta_{13}) = 0.02$, the energy resolution shall be better than $3\%\sqrt{E_{\rm vis}}$. This is actually a very stringent requirement, at least a factor of 2 better than that of the existing reactor neutrino experiments. For a typical liquid scintillator experiment, substantially more light shall be collected to reach such a level.

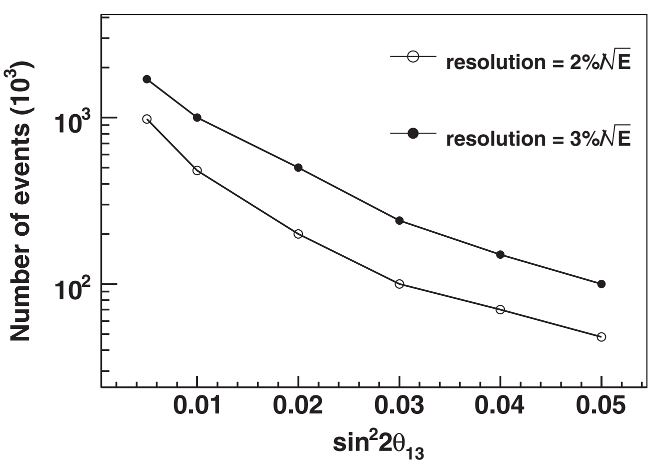


FIG. 6. Requirements to the number of events to determine the mass hierarchy at 90% probability as a function of $\sin^2(2\theta_{13})$.

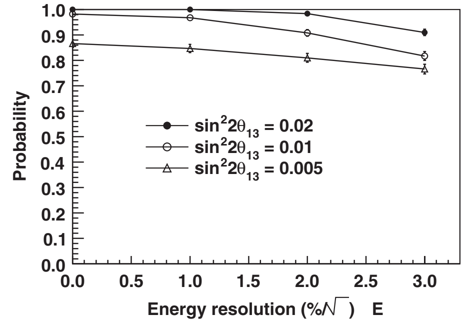


FIG. 7. Impact of the energy resolution to the determination probability for $\sin^2(2\theta_{13}) = 0.02$, 0.01, and 0.005.

Figure 8 shows the impact of $\sin^2(2\theta_{13})$ to the determination probability in four cases of the energy resolution $\{0/\sqrt{E_{\rm vis}}, 1\%/\sqrt{E_{\rm vis}}, 2\%/\sqrt{E_{\rm vis}}, 3\%/\sqrt{E_{\rm vis}}\}$. It shows that the energy resolution is very important to determine the mass hierarchy, while a larger $\sin^2(2\theta_{13})$ can relax substantially such a requirement.

The impact of the energy scale uncertainty is studied by transforming the sampled neutrino energy E to $E' = (1 + a)E + b$. For two cases of $a = -1\%$ or $b = -0.01$ MeV, which correspond to the shrinking or left-shift of the neutrino energy spectrum, the FCT spectra are calculated and shown in Fig. 9. It shows that the FCT spectra, both for NH and IH cases, are left shifted. After shrinking the energy spectrum, the L/E spectrum expands and the oscillation frequency becomes smaller, which results in the frequency spectra (FCT spectra) left shifted. The energy scale uncertainty only introduces a bias to the oscillation frequency and hence Δm^2_{31} (shown as the main peak in the FCT spectrum). Since our method only depends on the relative position of peaks or valleys in FCT and FST spectra, the mass hierarchy determination is not affected by the energy scale uncertainty.

To estimate the impact of oscillation parameters, we simulate experiments by scanning the oscillation parameters in their 3σ ranges [11]. When we scan one oscillation parameter, the other parameters are kept at their default

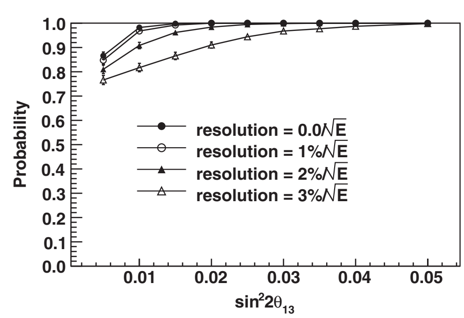


FIG. 8. Impact of $\sin^2(2\theta_{13})$ to the determination probability in four cases of the energy resolution.

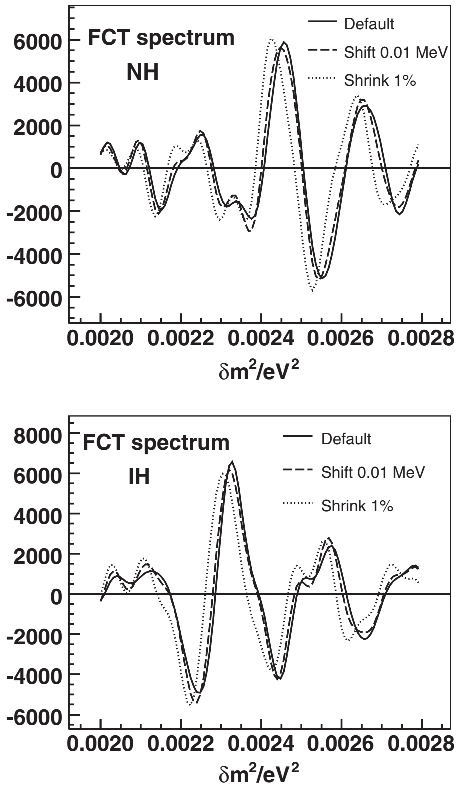


FIG. 9. FCT spectra for an energy shift of 0.01 MeV and shrinking of 1%.

values shown in Table I. We can see the Δm^2_{21}, $|\Delta m^2_{32}|$, and $\sin^2\theta_{12}$ only change the determination probability by several percents shown in Table II. Larger Δm^2_{21} and/or smaller $|\Delta m^2_{32}|$ increase the value of $\frac{\Delta m^2_{21}}{|\Delta m^2_{32}|}$, thus the features of the mass hierarchy will be more distinct. The $\sin^2\theta_{12}$ impacts the intensity of P_{32} oscillation, which can be seen from Eq. (2), thus larger $\sin^2\theta_{12}$ slightly increases the determination probability. The baseline has a larger impact compared with the oscillation parameters because it impacts all oscillation components of P_{21}, P_{32}, and P_{31}.

TABLE II. Determination probability variations are shown when scanning the oscillation parameters in their 3σ ranges. The default determination probability is 91%.

	$\Delta m^2_{21}/10^{-5}$ eV^2	$|\Delta m^2_{32}|/10^{-3}$ eV^2	$\sin^2\theta_{12}$
3σ range	7.14–8.19	2.06–2.81	2.06–0.375
Det. Prob.	88%–94.5%	92%–86%	85%–94%

In summary, we have studied experimental requirements to determine the mass hierarchy using Fourier cosine and sine transform to the reactor neutrino L/E spectrum. The parameters RL and PV are defined to extract features of the Fourier sine and cosine spectra, and the mass hierarchy can be determined from events collected in experiments similar to that in the analytical case. The impacts of baseline, event statistics, energy resolution, and energy scale uncertainty to the mass hierarchy determination are studied in detail. This paper provides a guidance to the design of the experiment to determine the mass hierarchy using reactor neutrinos.

[1] O. Mena, S. Palomares-Ruiz, and S. Pascoli, Phys. Rev. D **73**, 073007 (2006).
[2] D. S. Ayres *et al.* (NOvA Collaboration), arXiv:hep-ex/0503053.
[3] O. Mena, H. Nunokawa, and S. J. Parke, Phys. Rev. D **75**, 033002 (2007).
[4] K. Hagiwara, N. Okamura, and K. I. Senda, Phys. Rev. D **76**, 093002 (2007).
[5] S. T. Petcov and M. Piai, Phys. Lett. B **533**, 94 (2002).
[6] S. Choubey, S. T. Petcov, and M. Piai, Phys. Rev. D **68**, 113006 (2003).
[7] J. G. Learned, S. T. Dye, S. Pakvasa, and R. C. Svoboda, Phys. Rev. D **78**, 071302(R) (2008).
[8] L. Zhan, Y. Wang, J. Cao, and L. Wen, Phys. Rev. D **78**, 111103(R) (2008).
[9] S. M. Bilenky, D. Nicolo, and S. T. Petcov, Phys. Lett. B **538**, 77 (2002).
[10] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo, and A. M. Rotunno, Phys. Rev. Lett. **101**, 141801 (2008).
[11] G. L. Fogli *et al.*, Phys. Rev. D **78**, 033010 (2008).